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The Mathematics behind Integral Calculus

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ABSTRACT: Calculus **i**s that branch of Mathematics which is concerned with the calculation of instantaneous rates of change (Differential Calculus) and the summation of infinitely many small factors to determine some whole (Integral Calculus). Two mathematicians, Isaac Newton of England and Gottfried Wilhelm Leibniz of Germany, share credit for having independently developed the calculus in the 17th century. Calculus is now the basic entry point for anyone wishing to study physics, chemistry, biology, economics, finance, or actuarial science. Calculus makes it possible to solve problems as diverse as tracking the position of a space shuttle or predicting the pressure building up behind a dam as the water rises. Computers have become a valuable tool for solving calculus problems that were once considered impossibly difficult.

KEYWORDS: Calculus, infinitesimal, limit, continuity, derivative.

I.INTRODUCTION

Integral calculus, Branch of calculus concerned with the theory and applications of integrals. While differential calculus focuses on rates of change, such as slopes of tangent lines and velocities, integral calculus deals with total size or value, such as lengths, areas, and volumes. The two branches are connected by the fundamental theorem of calculus, which shows how a definite integral is calculated by using its antiderivative (a function whose rate of change, or derivative, equals the function being integrated). For example, integrating a velocity function yields a distance function, which enables the distance travelled by an object over an interval of time to be calculated. As a result, much of integral calculus deals with the derivation of formulas for finding antiderivatives. The great utility of the subject emanates from its use in solving differential equations.

1.2 Integration and Differentiation

Independently, Newton and Leibniz established simple rules for finding the formula for the slope of the tangent to a curve at any point on it, given only a formula for the curve. The rate of change of a function f (denoted by f') is known as its derivative. Finding the formula of the derivative function is called differentiation, and the rules for doing so form the basis of differential calculus. Depending on the context, derivatives may be interpreted as slopes of tangent lines, velocities of moving particles, or other quantities, and therein lies the great power of the differential calculus.

An important application of differential calculus is graphing a curve given its equation$y = f(x)$. This involves, in particular, finding local maximum and minimum points on the graph, as well as changes in inflection (convex to concave, or vice versa). When examining a function used in a mathematical model, such geometric notions have physical interpretations that allow a scientist or engineer to quickly gain a feeling for the behaviour of a physical system.

The other great discovery of Newton and Leibniz was that finding the derivatives of functions was, in a precise sense, the inverse of the problem of finding areas under curves—a principle now known as the fundamental theorem of calculus. Specifically, Newton discovered that if there exists a function $F(t)$ that denotes the area under the curve $y = f(x)$ from, say, 0 to t, then this function's derivative will equal the original curve over that interval, $F'(t) = f(t)$. Hence, to find the area under the curve $y = x^2$ from 0 to t, it is enough to find a function F so that $F'(t) = t^2$. The differential calculus shows that the most general such function is $x^3/3 + C$, where C is an arbitrary constant. This is called the (indefinite)integral of the function $y = x^2$, and it is written as $\int x^2 dx$. The initial symbol $\int$ is an elongated S, which stands for sum, and dx indicates an infinitely small increment of the variable, or axis, over which the function is being summed. Leibniz introduced this because he thought of integration as finding the area under a curve by a

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summation of the areas of infinitely many infinitesimally thin rectangles between the x-axis and the curve. Newton and Leibniz discovered that integrating $f(x)$ is equivalent to solving a differential equation—i.e., finding a function $F(t)$ so that $F'(t) = f(t)$. In physical terms, solving this equation can be interpreted as finding the distance $F(t)$ travelled by an object whose velocity has a given expression $f(t)$.

II. REVIEW OF RELATED LITERATURE:

2.1 Calculating curves and areas under curves

The roots of Calculus lie in some of the oldest geometry problems on record. The Egyptian Rhind papyrus (*c.* 1650 BCE) gives rules for finding the area of a circle and the volume of a truncated pyramid. Ancient Greek geometers investigated finding tangents to curves, the centre of gravity of plane and solid figures, and the volumes of objects formed by revolving various curves about a fixed axis.

By 1635 the Italian mathematician Bonaventura Cavalieri had supplemented the rigorous tools of Greek geometry with heuristic methods that used the idea of infinitely small segments of lines, areas, and volumes. In 1637 the French mathematician-philosopher René Descartes published his invention of analytic geometry for giving algebraic descriptions of geometric figures. Descartes's method, in combination with an ancient idea of curves being generated by a moving point, allowed mathematicians such as Newton to describe motion algebraically. Suddenly geometers could go beyond the single cases and ad hoc methods of previous times. They could see patterns of results, and so conjecture new results, that the older geometric language had obscured.

For example, the Greek geometer Archimedes (287–212/211 BCE) discovered as an isolated result that the area of a segment of a parabola is equal to a certain triangle. But with algebraic notation, in which a parabola is written as $y = x^2$, Cavalieri and other geometers soon noted that the area between this curve and the x-axis from 0 to a is $a^3/3$ and that a similar rule holds for the curve $y = x^3$—namely, that the corresponding area is $a^4/4$. From here it was not difficult for them to guess that the general formula for the area under a curve $y = x^n$ is $a^{n+1}/(n + 1)$.

2.2 Calculating velocities and slopes

The problem of finding tangents to curves was closely related to an important problem that arose from the Italian scientist Galileo Galilei's investigations of motion, that of finding the velocity at any instant of a particle moving according to some law. Galileo established that in t seconds a freely falling body falls a distance $gt^2/2$, where g is a constant (later interpreted by Newton as the gravitational constant). With the definition of average velocity as the distance per time, the body's average velocity over an interval from t to $t + h$ is given by the expression $[g(t + h)^2/2 - gt^2/2]/h$. This simplifies to $gt + gh/2$ and is called the difference quotient of the function $gt^2/2$. As h approaches 0, this formula approaches gt, which is interpreted as the instantaneous velocity of a falling body at time t.

This expression for motion is identical to that obtained for the slope of the tangent to the parabola $f(t) = y = gt^2/2$ at the point t. In this geometric context, the expression $gt + gh/2$ (or its equivalent $[f(t + h) - f(t)]/h$) denotes the slope of a secant line connecting the point $(t, f(t))$ to the nearby point $(t + h, f(t + h))$ (*see* figure). In the limit, with smaller and smaller intervals h, the secant line approaches the tangent line and its slope at the point t.

Thus, the difference quotient can be interpreted as instantaneous velocity or as the slope of a tangent to a curve. It was the calculus that established this deep connection between geometry and physics—in the process transforming physics and giving a new impetus to the study of geometry.

Integration, in Mathematics, technique of finding a function $g(x)$ the derivative of which, $Dg(x)$, is equal to a given function $f(x)$. This is indicated by the integral sign "∫," as in $\int f(x)$, usually called the indefinite integral of the function. The symbol dx represents an infinitesimal displacement along x; thus $\int f(x)dx$ is the summation of the product of $f(x)$ and dx. The definite integral, written $\int_a^b f(x)\,dx$ with a and b called the limits of integration, is equal to $g(b) - g(a)$, where $Dg(x) = f(x)$.

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Some antiderivatives can be calculated by merely recalling which function has a given derivative, but the techniques of integration mostly involve classifying the functions according to which types of manipulations will change the function into a form the antiderivative of which can be more easily recognized. For example, if one is familiar with derivatives, the function $1/(x + 1)$ can be easily recognized as the derivative of $\log_e(x + 1)$. The antiderivative of $(x^2 + x + 1)/(x + 1)$ cannot be so easily recognized, but if written as $x(x + 1)/(x + 1) + 1/(x + 1) = x + 1/(x + 1)$, it then can be recognized as the derivative of $x^2/2 + \log_e(x + 1)$. One useful aid for integration is the theorem known as integration by parts. In symbols, the rule is $\int fDg = fg - \int gDf$. That is, if a function is the product of two other functions, f and one that can be recognized as the derivative of some function g, then the original problem can be solved if one can integrate the product gDf. For example, if $f = x$, and $Dg = \cos x$, then $\int x{\cdot}\cos x = x{\cdot}\sin x - \int \sin x = x{\cdot}\sin x - \cos x + C$. Integrals are used to evaluate such quantities as area, volume, work, and, in general, any quantity that can be interpreted as the area under a curve.

2.3 Length of a curve

Calculating the length of a curve, geometrical concept addressed by integral calculus. Methods for calculating exact lengths of line segments and arcs of circles have been known since ancient times. Analytic geometry allowed them to be stated as formulas involving coordinates (coordinate systems) of points and measurements of angles. Calculus provided a way to find the length of a curve by breaking it into smaller and smaller line segments or arcs of circles. The exact value of a curve's length is found by combining such a process with the idea of a limit. The entire procedure is summarized by a formula involving the integral of the function describing the curve.

III. FUNDAMENTAL THEOREM OF CALCULUS

Basic principle of calculusrelates the derivative to the integral and provides the principal method for evaluating definite integrals. In brief, it states that any function that is continuous (continuity) over an interval has an antiderivative (a function whose rate of change, or derivative, equals the function) on that interval. Further, the definite integral of such a function over an interval $a<x<b$ is the difference $F(b) - F(a)$, where F is an antiderivative of the function. This particularly elegant theorem shows the inverse function relationship of the derivative and the integral and serves as the backbone of the physical sciences. It was articulated independently by Isaac Newton and Gottfried Wilhelm Leibniz.

In this paper, study is carried out on the properties of Integral Calculus strictly.

3.1 Integral Calculus

Integral calculus helps in finding the anti-derivatives of a function. These anti-derivatives are also called the integrals of the function. The process of finding the anti-derivative of a function is called integration. The inverse process of finding derivatives is finding the integrals. The integral of a function represents a family of curves. Finding both derivatives and integrals form the fundamental calculus. In paper covers the basics of integrals and evaluating integrals.

Integrals are the values of the function found by the process of integration. The process of getting f(x) from f'(x) is called integration. Integrals assign numbers to functions in a way that describe displacement and motion problems, area and volume problems, and so on that arise by combining all the small data. Given the derivative f' of the function f, we can determine the function f. Here, the function f is called antiderivative or integral of f'.

Example: Given: $f(x) = x^2$

Derivative of $f(x) = f'(x) = 2x = g(x)$

if $g(x) = 2x$, then anti-derivative of $g(x) = \int g(x) = x^2$

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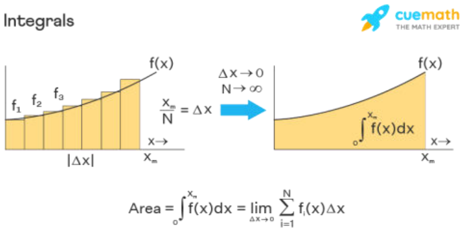
Definition of Integral

F(x) is called an antiderivative or Newton-Leibnitz integral or primitive of a function f(x) on an interval I. F'(x) = f(x), for every value of x in I.

Integral is the representation of the area of a region under a curve. We approximate the actual value of an integral by drawing rectangles. A definite integral of a function can be represented as the area of the region bounded by its graph of the given function between two points in the line. The area of a region is found by breaking it into thin vertical rectangles and applying the lower and the upper limits, the area of the region is summed up. We specify an integral of a function over an interval on which the integral is defined.



3.3 Fundamental Theorems of Integral Calculus

We define integrals as the function of the area bounded by the curve y = f(x), a ≤ x ≤ b, the x-axis, and the ordinates x = a and x =b, where b>a. Let x be a given point in [a,b]. Then $\int_a^b f(x)dx$represents the area function. This concept of area function leads to the fundamental theorems of integral calculus.

- First Fundamental Theorem of Integral Calculus
- Second Fundamental Theorem of Integral Calculus

3.4 First Fundamental Theorem of Integrals

A(x) =$\int_a^b f(x)dx$, for all x ≥ a, where the function is continuous on [a,b]. Then A'(x) = f(x) for all x ϵ [a,b]

3.5 Second Fundamental Theorem of Integrals

If f is continuous function of x defined on the closed interval [a,b] and F be another function such that d/dx F(x) = f(x) for all x in the domain of f, then $\int_a^b f(x)dx$= f(b) -f(a). This is known as the definite integral of the function f over the range [a,b], a being the lower limit and b the upper limit.

3.6 Types of Integrals

Integral calculus is used for solving the problems of the following types.

1) The problem of finding a function if its derivative is given.

2) The problem of finding the area bounded by the graph of a function under given conditions. Thus the Integral calculus is divided into two types.

a) Definite Integrals (the value of the integrals is definite)
b) Indefinite Integrals (the value of the integral is indefinite with an arbitrary constant, C)

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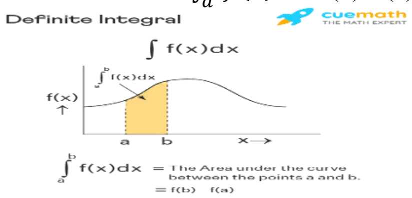
3.7 Indefinite Integrals

These are the integrals that do not have a pre-existing value of limits; thus making the final value of integral indefinite. ∫g'(x)dx = g(x) + c. Indefinite integrals belong to the family of parallel curves.

3.8 Definite Integrals

The definite integrals have a pre-existing value of limits, thus making the final value of an integral, definite. if f(x) is a function of the curve, then $\int_a^b f(x)dx$=f(b)−f(a)



3.9 Properties of Integral Calculus

Let us study the properties of indefinite integrals to work on them.

- The derivative of an integral is the integrand itself. ∫ f(x) dx = f(x) +C
- Two indefinite integrals with the same derivative lead to the same family of curves and so they are equivalent. ∫ [f(x) dx -g(x) dx] =0
- The integral of the sum or difference of a finite number of functions is equal to the sum or difference of the integrals of the individual functions. ∫ [f(x) dx+g(x) dx] = ∫ f(x) dx + ∫ g(x) dx
- The constant is taken outside the integral sign. ∫ k f(x) dx = k ∫ f(x) dx, where k ∈ R.
- The previous two properties are combined to get the form: ∫ [k1f1(x) + k2f2(x) +... knfn(x)] dx = k1∫ f1(x)dx + k2∫ f2(x)dx+ ... kn ∫ fn(x)dx

3.10 Integral Formulae

The following are some of the formulae of derivatives of some important functions. Here are the corresponding integrals of these functions that are remembered as standard formulas of integrals.

- $\int x^n dx = x^{n+1}/n+1+C$, where $n \neq -1$
- ∫ dx =x+C
- ∫ cosxdx = sinx+C
- ∫ sinx dx = -cosx+C
- $\int \sec^2 x\, dx = \tan x + C$
- $\int \mathrm{cosec}^2 x\, dx = -\cot x + C$
- $\int \sec^2 x\, dx = \tan x + C$
- ∫ secxtanxdx = secx+C
- ∫ cscxcotx dx = -cscx+C
- $\int 1/(\sqrt{(1-x^2)}) = \sin^{-1} x + C$
- $\int -1/(\sqrt{(1-x^2)}) = \cos^{-1} x + C$
- $\int 1/(1+x^2) = \tan^{-1} x + C$
- $\int -1/(1+x^2) = \cot^{-1} x + C$
- $\int 1/(x\sqrt{(x^2-1)}) = \sec^{-1} x + C$
- $\int -1/(x\sqrt{(x^2-1)}) = \mathrm{cosec}^{-1} x + C$
- $\int e^x dx = e^x + C$
- ∫dx/x=ln|x| + C
- $\int a^x dx = a^x/\ln a + C$

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3.11 Methods of Finding Integrals

There are several methods adopted for finding the indefinite integrals. The prominent methods are:

- Finding integrals by integration by substitution method
- Finding integrals by integration by parts
- Finding integrals by integration by partial fractions.

Finding Integrals by Substitution Method

A few integrals are found by the substitution method. If u is a function of x, then u' = du/dx.

$\int f(u)u'\,dx = \int f(u)du$, where $u = g(x)$.

3.12 Finding Integrals by Integration by Parts

If two functions are of the product form, integrals are found by the method of integration by parts.

$\int f(x)g(x)\,dx = f(x)\int g(x)\,dx - \int (f'(x) \int g(x)\,dx)\,dx$.

3.13 Finding Integrals by Integration by Partial Fractions

Integration of rational algebraic functions whose numerator and denominator contain positive integral powers of x with constant coefficients is done by resolving them into partial fractions.

To find $\int f(x)/g(x)\,dx$, decompose this improper rational function to a proper rational function and then integrate.

$\int f(x)/g(x)\,dx = \int p(x)/q(x) + \int r(x)/s(x)$, where $g(x) = a(x) . s(x)$

3.14 Applications of Integral Calculus

Using integration, we can find the distance given the velocity. Definite integrals form the powerful tool to find the area under simple curves, the area bounded by a curve and a line, the area between two curves, the volume of the solids. The displacement and motion problems also find their applications of integrals. The area of the region enclosed between two curves y = f(x) and y = g(x) and the lines x =a, x =b is given by

Area = $\int_a^b [(f(x) - g(x))]dx$

Let us find the area bounded by the curve y = x and y = x^2 that intersect at (0,0)and (1,1).

The given curves are that of a line and a parabola. The area bounded by the curves = $\int_0^1 (y2 - y1)dx$

Area = $\int_0^1 (x - x2)dx = x^2/2 - x^3/3 = 1/2 - 1/3 = 1/6$ square units.

- The primitive value of the function found by the process of integration is called an integral.
- An integral is a mathematical object that can be interpreted as an area or a generalization of area.
- When a polynomial function is integrated the degree of the integral increases by 1.

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3.15 Properties of Integrals

Properties of integrals define the rules for working across integral problems. The properties of integrals can be broadly classified into two types based on the type of integrals. They are the properties of indefinite integrals, and the properties of definite integrals.

The properties of integrals are helpful in solving integral problems. Integration involving functions and algebraic expressions. These problems can be solved with a thorough knowledge of the properties of integrals. The properties of integrals can be broadly classified as the following two types based on the types of integrals.

- Properties of indefinite integrals
- Properties of definite integrals.

3.16 Properties of Indefinite Integral

The following are the five important properties of indefinite integrals.

1. The process of integration and differentiation are reverse to each other. $\frac{d}{dx}\int f(x).dx=f(x)$ or $\int f'(x).dx=f(x)+C$, where C is an arbitrary constant.
2. Two indefinite integrals with the same derivative, if they are equal, then their function representing the family of curves are equivalent. $\frac{d}{dx}\int f(x).dx=\frac{d}{dx}\int g(x).dx$, then f(x) is equivalent to g(x).
3. The integration of the sum of two functions, is equal to the sum of the integration of the individual functions.

$\int[f(x)+g(x)].\ dx=\int f(x).\ dx+\int g(x).\ dx$

4. For a real number k, the integration of the product of k and the function is equal to the product of constant k and the integral of the function.

$\int k.f(x).dx=k\int f(x).dx$

The integration of the summation of the constant and the function is equal to the summation of the integrals of the product of the constant and the function. $\int[k1.f1(x)+k2.f2(x)+.....knfn(x)].dx=k1\int f1(x).dx+k2\int f2(x).dx+....kn\int fn(x).dx$

3.17 Discussion:

Calculus is the study of how things change. It provides a framework for modeling systems in which there is change, and a way to deduce the predictions of such models.The branch of the calculus concerned with calculating integrals is the integral calculus, and among its many applications, it is applied in finding work done by physical systems and calculating pressure behind a dam at a given depth.Calculus is of outmost importance because of its huge applicability. Calculus is not restricted to mathematics and analysis, it's used pretty much everywhere - Physics, Chemistry, Economics, Biology, Engineering, Dynamic systems and so much more. That's where the importance of calculus comes from.

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